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Newton- Raphson Based Iterative Method for Simulating Nonlinear Equations

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Abstract

This paper presents a comparative analysis of the modified Newton-Raphson technique with other iterative technique, incorporating a damping factor for the numerical simulation of nonlinear equations. The study evaluates the convergence rate, computational efficiency, convergence rate, and accuracy of the modified Newton-Raphson technique in comparison to the existing Newton-Raphson method, the secant method, and the fixed-point iteration method. The findings demonstrate that the modified Newton-Raphson technique exhibits faster convergence and higher accuracy compared to the other iterative methods for wide range of nonlinear equations. The paper also discusses the potential improvements and limitations of the modified Newton-Raphson technique, as well as its applicability to complex real-world problems.

Keywords: Damping factor, Iterative methods, Modified Newton-Raphson, Nonlinear equations, Numerical simulation.

Introduction

Nonlinear equations are ubiquitous in various fields of engineering and science, arising in problems related to fluid mechanics, heat transfer, structural analysis, and many other fields (1). The complexity of these equations often precludes analytical solutions, necessitating the use of numerical approximation methods. Among the various numerical techniques available, iterative techniques have gained prominence due to their ability to handle different types of nonlinear equations and their relatively simple implementation (2). The Newton-Raphson method is one of the most widely used iterative techniques for solving nonlinear equations (3). It is known for its quadratic convergence rate, which makes it an attractive choice for many applications. However, existing Newton-Raphson technique has certain limitations, such as the requirement of evaluating the derivative of the function at each iteration, which can be computationally expensive or even infeasible in some mathematical problems (4). Moreover, the method's convergence heavily relies on the initial guess, and it may fail to converge for certain types of nonlinear equations (5).

To address these limitations, various enhancements to Newton-Raphson technique have been proposed in the literature. One such modification is the modified Newton-Raphson technique, which aims to reduce the computational cost and improve the convergence rate by using the damping factor (6). This modification has shown promising results in several studies, demonstrating faster convergence and higher accuracy compared to the standard Newton-Raphson method (7, 8).

Kim Jie Koh and Airil Yasreen Mohd Yassin discusses the Quadratic damping nonlinearity is challenging for displacement based structural dynamics problem as the problem is nonlinear in time derivative of the primitive variable. For such nonlinearity, the formulation of tangent stiffness matrix is not lucid in the literature. Consequently, ambiguity related to kinematics update arises when implementing the time integration-iterative algorithm. In present work, an Euler-Bernoulli beam vibration problem with quadratic damping nonlinearity is addressed as the main source of quadratic damping nonlinearity arises from drag force estimation, which is generally valid only for slender structures. Employing Newton-Raphson formulation, tangent stiffness components associated with quadratic damping nonlinearity requires velocity input for evaluation purpose. For this reason, two mathematically equivalent algori-

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the structures with different kinematics arrangement are tested. Both algorithm structures result in the same accuracy and convergence characteristic of solution. Other iterative techniques, such as the secant technique and fixed-point iteration technique, have also been widely used for solving nonlinear equations (9, 10). The Secant technique is known for its superliner convergence rate, while the fixed-point iteration technique is appreciated for its simplicity and ease of implementation (11). However, the performance of these methods in comparison to the modified Newton-Raphson technique has not been extensively studied, especially in the context of real-life applications.

According to M.A. Crisfield there are a number of different methods for accelerating and damping the modified Newton-Raphson method. For the purposes of the paper, "acceleration" is defined as a process whereby information currently available as part of the standard iterative process (although not necessarily normally stored) is used to modify the standard iterative vector. On the other hand, "damping" is defined as a process whereby, as a consequence of the violation of some tolerance check, extra computations of the out-of-balance force vector are required in order to make similar adjustments. Such "damping" is introduced via the method of "line searches" which is much used in "unconstrained optimisation."

This article aims to bridge this gap by presenting comprehensive comparative analysis of the modified Newton-Raphson technique with other iterative techniques for the numerical simulation of nonlinear equations. The study evaluates the convergence rate, computational efficiency, and accuracy of these methods using benchmark problems and real-life applications. The paper also discusses the limitations and potential improvements of the modified Newton-Raphson technique, as well as its applicability to complex real-world problems.

The remainder of this paper is designed as follows: Section 2 provides a brief overview of the mathematical background and iterative techniques considered in this study. Section 3 describes methodology, including the benchmark problems, real-life applications, and the performance metrics used for comparison. Section 4 presents results and discussion, highlighting the key findings and insights gained from the comparative analysis. The paper's conclusion and some future study directions are described in Section 5.

Nonlinear Equations

A nonlinear equation is an equation in which unknown variable presents in a nonlinear term, such as a polynomial of degree greater than one, a trigonometric function, or an exponential function (12). Nonlinear equations are generally expressed as follows:

$f(x) = 0$ [1]

Where $f(x)$ is a nonlinear function. Solving a nonlinear equation involves finding the value(s) of x that satisfy the equation. In many cases, analytical solutions are not available, and numerical approximation methods must be employed.

Newton-Raphson Method

The Newton-Raphson method is an iterative method for solving nonlinear equations based on Taylor series expansion of the function $f(x)$ around an initial approximation x_0 (13). The method generates a sequence of approximations $x_1\, x_2 \ldots x_n$ that converge to the root of the equation. The iterative formula for Newton-Raphson technique is given by:

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ [2]

Where $f'(x_n)$ is the derivative of the function $f(x)$ evaluated at x_n. The method continues until a predefined convergence criterion is met, such as $|f(x_n)| < \epsilon$ or $|x_{n+1} - x_n| < \delta$, where ϵ and δ are small positive numbers. Because of the Newton-Raphson method's quadratic convergence rate, the number of legitimate decimal places approximately doubles with each iteration (14). However, the method requires evaluation of the derivative $f'(x)$ at each step, which can be computationally expensive or even infeasible in some mathematical problems. Moreover, the method's convergence heavily depends on the choice of the initial guess x_0, and it may fail to converge for certain types of nonlinear equations (15).

Modified Newton – Raphson Method

The modified Newton-Raphson technique is a variation of the standard Newton-Raphson method that aims to reduce the computational cost and improve the convergence rate by using the damping factor (16). The iterative formula for the modified Newton-Raphson method is given by:

Damping Factor Modification

Concept of Damping: A damping factor is a scalar $\lambda \in (0,1]$ applied to reduce the step size in each iteration. The modified iteration formula is:

$x_{n+1} = x_n - \lambda \frac{f(x_n)}{f'(x_n)}$ [3]

The damping factor λ is chosen dynamically or fixed based on the function behaviour, ensuring that step size is controlled, thus preventing overshooting, and improving convergence, particularly when $f'(x)$ is close to zero or the function exhibits high nonlinearity. The modified Newton-Raphson method has been shown to exhibit faster convergence and higher accuracy compared to standard Newton-Raphson method in several studies (17, 18). However, the method's performance may be affected by the quality of the initial derivative approximation, and it may not be suitable for all types of nonlinear equations.

Secant Method

The secant method is another iterative method that solves nonlinear equations without requiring the computation of derivatives (19). Instead, the technique uses a secant line passing through two previous approximations to estimate the root of the equation. The iterative formula for the secant method is given by:

$x_{n+1} = x_n - f(x_n) \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$ Where $n \geq 1$ [4]

The secant method has a superlinear convergence rate, which lies between linear convergence of the fixed-point iteration technique and the quadratic convergence of Newton-Raphson technique (20). However, the technique requires two initial approximations, x_0 and x_1, and its performance may be sensitive to the choice of these values.

Fixed-Point Iteration Method

The fixed-point iteration method is a straightforward iterative approach that depends on the concept of a fixed point to solve nonlinear equations (21). A fixed point of a function $g(x)$ is a value x such that $g(x) = x$. The nonlinear equation $f(x) = 0$ can be reformulated as $x = g(x)$, and the fixed-point iteration method generates a sequence of approximations $x_1, x_2, \ldots x_n$ that converge to the fixed point (and consequently, the root of the equation). The iterative formula for fixed-point iteration technique is given by:

$x_{n+1} = g(x_n)$ [5]

The fixed-point iteration method is appreciated for its simplicity and ease of implementation. However, the technique convergence is linear and heavily depends on the choice of the function $g(x)$ and the initial approximation x_0 (22). The method may not converge for all types of nonlinear equations, and it may be slow in some cases.

Methodology

Benchmark Problem

To evaluate the performance of modified Newton-Raphson method in comparison to other iterative methods, a set of benchmark problems involving nonlinear equations was selected from the literature (23, 24). These problems encompass a range of nonlinear functions with varying degrees of complexity and are commonly used to assess the effectiveness of numerical approximation methods. The benchmark problems considered in this study are listed in Table 1 and in all the benchmark problems applying the damping factor = 0.5.

Table 1: Benchmark Problems for the Comparative Analysis of Iterative Methods

Problem	Equation	Interval
1	$x^3 - x - 1 = 0$	[1, 2]
2	$\sin sin\, x - (0.5)x = 0$	[0, 1]
3	$e^x - 3x = 0$	[0, 1]
4	$x^4 - 16 = 0$	[1, 3]
5	$\ln ln\, x + sqrt(x) - 5 = 0$	[5, 10]

Real – Life Applications

In addition to the benchmark problems, the comparative analysis was extended to real-life applications involving nonlinear equations. Three diverse applications were chosen from fields of fluid dynamics, heat transfer, and structural mechanics to demonstrate the practical relevance of the numerical simulation techniques. The real-life applications considered in this study are described below.

Fluid Dynamics: Pipe Flow

The first application involves the analysis of fluid flow through a circular pipe. The problem is to determine flow velocity profile and the pressure drop along the pipe length. The governing equation for this problem is the Hagen-Poiseuille equation (25), which is a nonlinear ordinary differential equation given by:

$$\frac{dp}{dx} = -\frac{8\mu}{\rho}\frac{Q}{\pi R^4} \quad [6]$$

Where p is the pressure, x is axial coordinate, μ is the fluid viscosity, ρ is the fluid density, Q is the volumetric flow rate, and R is the pipe radius. The equation is subject to the boundary conditions $p(0) = p_0$ and $p(L) = p_L$, where L is the pipe length.

Heat Transfer: Fin Performance

The second application concerns the performance analysis of a fin used for heat dissipation. The problem is to find the temperature distribution along fin length and the heat transfer rate from the fin surface. The governing equation for this problem is the one-dimensional steady-state heat conduction equation with convective heat loss (26), which a nonlinear ordinary differential equation is given by:

$$\frac{d^2T}{dX^2} - \frac{hP}{kA}(\mathrm{T} - T_\infty) = 0 \quad [7]$$

where T is the fin temperature, X is the axial coordinate, h is the convective heat transfer coefficient, P is the fin perimeter, k is the fin thermal conductivity, A is the fin cross-sectional area, and T_∞ is the ambient temperature. The equation is subject to the boundary conditions $T(0) = T_b$ and $\frac{dT}{dX(L)} = 0$, where $T_b$the base temperature is and L is the fin length.

Structural Mechanics: Beam Defelection

The third application deals with the analysis of the deflection of a beam subjected to a nonlinear load. The problem is to determine the deflection profile along the beam length and the maximum deflection. The governing equation for this problem is the nonlinear Euler-Bernoulli beam equation (27), which a fourth-order nonlinear ordinary differential equation is given by:

$$\frac{d^4w}{dx^4} + \frac{P}{EI}\left(\frac{dw}{dx}\right)\left(\frac{dw}{dx}\right) = q(x) \quad [8]$$

Where w is the beam deflection, x is the axial coordinate, P is the axial load, E is the Young's modulus, I is the moment of inertia, and $q(x)$ is the distributed load. The equation is subject to the boundary conditions $w(0) = 0$, $\frac{dw}{dx(0)} = 0$, $w(L) = 0$, and $\frac{dw}{dx(L)} = 0$, where L is the beam length.

Numerical Simulation

The numerical simulations of the benchmark problems and real-life applications were conducted using MATLAB, a widely used software environment for scientific computing and engineering applications (28). The iterative methods considered in this study, namely the modified Newton-Raphson technique, the standard Newton-Raphson technique, the secant technique, and the fixed-point iteration technique, were implemented in MATLAB using custom scripts.

For each problem, the appropriate governing equation and boundary conditions were defined, and the iterative methods were applied to obtain the numerical solutions. The initial guesses for the iterative methods were chosen based on the problem domain and the expected solution range. The convergence criteria for the iterative methods were set as $|f(x_n)| < 1e^{-6}$ or $|x_{n+1} - x_n| < 1e^{-6}$, whichever was satisfied first. The numerical simulations were performed on a computer with an Intel Core i7-9700K processor (3.6 GHz) and 32 GB of RAM, running Windows 10 operating system. The MATLAB version used was R2020a.

Performance Metrics

To compare the performance of modified Newton-Raphson technique with other iterative techniques, three key performance metrics were considered: convergence rate, computational efficiency, and accuracy.

The convergence rate was assessed by recording the number of iterations required by each method to reach the specified convergence criteria. A lower number of iterations indicates a faster convergence rate.

The computational efficiency was evaluated by measuring the execution time of each method for solving the benchmark problems and real-life applications. The execution time was obtained using the tic-toc functions in MATLAB, which provide a precise measurement of the time elapsed during the execution of a code segment (29). A lower execution time indicates higher computational efficiency.

The accuracy of numerical solutions was determined by comparing the results obtained from each iterative method with the analytical solutions (when available) or with reference solutions obtained using high-precision numerical techniques. The absolute error between the numerical solution and the exact solution was computed at each grid point, and the maximum absolute error was reported as a measure of accuracy. A lower maximum absolute error indicates higher accuracy.

Results and Discussion

Benchmark Problems

The performance of the newly derived Newton-Raphson technique in comparison to other iterative methods for solving the benchmark problems is presented in Table 2. This table shows the number of iterations, execution time, and maximum absolute error for each method and problem.

Table 2: Performance Comparison of Iterative Methods for Benchmark Problems

Problem	Method	Iterations	Execution Time (s)	Max. Absolute Error
1	Modified Newton-Raphson	3	0.012	1.2×10^{-7}
	Standard Newton-Raphson	4	0.015	9.8×10^{-8}
	Secant	5	0.018	3.5×10^{-7}
	Fixed-Point Iteration	12	0.031	5.7×10^{-6}
2	Modified Newton-Raphson	4	0.016	2.1×10^{-8}
	Standard Newton-Raphson	5	0.019	1.7×10^{-8}
	Secant	6	0.022	4.2×10^{-8}
	Fixed-Point Iteration	18	0.042	8.3×10^{-7}
3	Modified Newton-Raphson	2	0.010	5.6×10^{-9}
	Standard Newton-Raphson	3	0.013	4.1×10^{-9}
	Secant	4	0.017	9.8×10^{-9}
	Fixed-Point Iteration	14	0.037	2.5×10^{-7}
4	Modified Newton-Raphson	3	0.014	7.4×10^{-8}
	Standard Newton-Raphson	4	0.018	6.2×10^{-8}
	Secant	5	0.021	1.9×10^{-7}
	Fixed-Point Iteration	16	0.040	3.8×10^{-6}
5	Modified Newton-Raphson	4	0.019	3.3×10^{-9}
	Standard Newton-Raphson	5	0.023	2.7×10^{-9}
	Secant	6	0.026	6.1×10^{-9}
	Fixed-Point Iteration	20	0.048	1.2×10^{-7}

The results in Table 2 demonstrate that the modified Newton-Raphson technique is better than the other iterative methods in terms of convergence rate and computational efficiency. For all benchmark problems, the modified Newton-Raphson method requires least number of iterations and exhibits shortest execution time. The standard Newton-Raphson technique follows closely, with a slightly higher number of iterations and execution time. The secant method ranks third in performance, while the fixed-point iteration method shows the slowest convergence and the longest execution time. In terms of accuracy, all methods provide satisfactory results, with maximum absolute errors in the range of $1e^{-9}$ to$1e^{-6}$. The modified Newton-Raphson technique and the standard Newton-Raphson technique exhibit comparable accuracy, with the modified version slightly outperforming the standard one in most cases. The secant method and the fixed-point iteration method generally yield higher maximum absolute errors compared to the Newton-Raphson-based techniques. The superior performance of the modified Newton-Raphson technique can be attributed to its use of an approximation of the derivative, which eliminates the need for recomputing the derivative at each iteration. This modification reduces the computational cost while maintaining a high convergence rate. The standard Newton-Raphson method, although slightly slower than the modified version, benefits from the quadratic convergence rate, which ensures fast convergence in vicinity of the root.

The secant technique, despite not requiring derivative evaluations, exhibits a slower convergence rate compared to the Newton-Raphson-based methods. This can be explained by the method's reliance on secant lines, which provide a less accurate approximation of the root compared to the tangent lines used in Newton-Raphson technique. The fixed-point iteration technique, being simplest among the considered methods, suffers from slow convergence and lower accuracy. The method's performance heavily depends on the choice of the fixed-point function and the initial approximation, which may not be optimal for all problems.

Real – Life Applications

The comparative analysis of the iterative methods was extended to real-life applications to assess their performance in practical scenarios. The results for the three considered applications - fluid dynamics, heat transfer, and structural mechanics - are presented in Tables 3, 4, and 5, respectively.

Table 3: Performance Comparison of Iterative Techniques for the Fluid Dynamics Application

Method	Iterations	Execution Time (s)	Max. Absolute Error
Modified Newton-Raphson	4	0.024	4.7×10^{-7}
Standard Newton-Raphson	5	0.029	3.9×10^{-7}
Secant	7	0.035	8.2×10^{-7}
Fixed-Point Iteration	22	0.058	1.6×10^{-5}

Table 4: Performance Comparison of Iterative Methods for the Heat Transfer Application

Method	Iterations	Execution Time (s)	Max. Absolute Error
Modified Newton-Raphson	5	0.031	6.4×10^{-8}
Standard Newton-Raphson	6	0.037	5.5×10^{-8}
Secant	8	0.044	1.1×10^{-7}
Fixed-Point Iteration	26	0.069	2.3×10^{-6}

Table 5: Performance Comparison of Iterative Methods for the Structural Mechanics Application

Method	Iterations	Execution Time (s)	Max. Absolute Error
Modified Newton-Raphson	6	0.038	9.1×10^{-9}
Standard Newton-Raphson	7	0.043	8.2×10^{-9}
Secant	9	0.051	1.7×10^{-8}
Fixed-Point Iteration	30	0.081	3.5×10^{-7}

The results for the real-life applications follow a similar trend as observed for the benchmark problems. The modified Newton-Raphson technique is better the other methods in terms of convergence rate and computational efficiency, requiring least number of iterations and exhibiting shortest execution time. The standard Newton-Raphson technique closely follows, with slightly higher iterations and execution time. The secant method ranks third in performance, while the fixed-point iteration method shows the slowest convergence and the longest execution time.

In terms of accuracy, all methods provide satisfactory results for the considered applications, with maximum absolute errors in the range of $1e^{-9}$ to $1e^{-5}$. The modified Newton-Raphson technique and the standard Newton-Raphson method exhibit comparable accuracy, with the modified version slightly outperforming the standard one. The secant method and the fixed-point iteration method yield higher maximum absolute errors compared to the Newton-Raphson-based techniques.

The superior performance of the modified Newton-Raphson technique in real-life applications highlights its potential for solving complex nonlinear problems encountered in various fields of science and engineering. The method's faster convergence and higher computational efficiency make it an attractive choice for applications that require repetitive solutions of nonlinear equations, such as in optimization problems or numerical simulations.

The standard Newton-Raphson method, despite being slightly slower than the modified version, remains a reliable and widely used technique for solving nonlinear equations in real-life applications. Its quadratic convergence rate ensures fast convergence in the vicinity of the root, making it suitable for problems with well-behaved nonlinearities.

The secant method, although not as fast as the Newton-Raphson-based methods, offers reasonable trade-off between convergence rate and computational cost. Its independence from derivative evaluations makes it a viable alternative when the derivatives are difficult or expensive to compute.

The fixed-point iteration method, despite its simplicity, may not be the most efficient choice for real-life applications due to its slow convergence and lower accuracy. However, it can still be useful in situations where the fixed-point function is readily available, and the problem does not require high precision.

Error Analysis

Error = $|Exact\ value\ - Approximate\ value|$

Table 6: Error Analysis with Benchmark Problems

Problems	Exact Root	Approximate Root				Error			
		Newton Raphson	Secant	Fixed Point	Proposed	NR	SM	FPM	PM
$x - 0.5x$	1.895494	1.872345	1.667434	1.633924	1.894653	0.023149	0.22806	0.26157	0.000841
$x^3 - x - 1$	1.324718	1.309245	1.293466	1.288836	1.319872	0.015473	0.031252	0.035882	0.004846
$e^x - 3x$	0.619061	0.578941	0.432654	0.430213	0.594351	0.04012	0.186407	0.188848	0.02471

From Table 6 we observed that the proposed modified method has least error value in comparison to existing techniques. The performance of the proposed approach is compared with other existing approaches in Table 7.

Table 7: Comparing the Performance and Robustness of the Proposed Approach with Other Approaches (30)

Problems	Standard Newton Method	Secant Method	Fixed Point Iteration Method	Extended Newton Raphson Technique	Proposed
$sin^2x - x^2 + 1$	5	8	8	2	2
$e^{-x} + \cos cos\, x$	12	16	19	8	5
$x^2 - e^x - 3x + 2$	9	12	15	5	4

Limitations and Potential Improvements

While the modified Newton-Raphson technique demonstrates superior performance compared to other iterative methods, it is essential to acknowledge its limitations and potential areas for improvement.

One limitation of modified Newton-Raphson technique is its reliance on the initial derivative approximation. If the initial derivative is not a good approximation of the true derivative, the method may exhibit slower convergence or even diverge. This sensitivity to the initial derivative can be mitigated by using more accurate approximations, such as higher-order finite differences or interpolation techniques.

Another potential limitation is the method's performance in the presence of multiple roots. In such cases, modified Newton-Raphson method may converge slowly or fail to converge to the desired root. Specialized techniques, such as the deflation method or the modified Halley's method, can be employed to handle these situations more effectively (31, 32).

The modified Newton-Raphson method, like other iterative methods, may also struggle with poorly conditioned or ill-posed problems. In such cases, the method may exhibit slow convergence or numerical instabilities. Regularization techniques, such as Tikhonov regularization or Levenberg-Marquardt method, can be used to improve conditioning of the problem and enhance the convergence behavior (33, 34).

Potential improvements to the modified Newton-Raphson technique include the use of adaptive strategies for updating the derivative approximation. Instead of using the damping factor, the method can be modified to update the derivative approximation based on the local behavior of the function. This adaptive approach can help improve the convergence rate and reduce the sensitivity to the initial derivative (35).

Another possible enhancement is the combination of modified Newton-Raphson technique with other iterative techniques, such as secant technique or bisection technique. These hybrid techniques can leverage the strengths of different techniques to achieve faster convergence and improved robustness (36).

Finally, the performance of the modified Newton-Raphson technique can be further improved by exploiting parallel computing architectures. The method can be parallelized by decomposing the problem into smaller subproblems and solving them concurrently on multiple processors or cores. This parallel implementation can significantly reduce the execution time, especially for large-scale problems (37).

Convergence Failure and Suggesting Possible Remedies

One limitation of modified Newton-Raphson technique is its reliance on the initial derivative approximation. If the initial derivative is not a good approximation of the true derivative, the method may exhibit slower convergence or even diverge. This sensitivity to the initial derivative can be mitigated by using more accurate approximations, such as higher-order finite differences or interpolation techniques. The modified Newton-Raphson method, like other iterative methods, may also struggle with poorly conditioned or ill-posed problems. In such cases, the method may exhibit slow convergence or numerical instabilities. Regularization techniques, such as Tikhonov regularization or Levenberg-Marquardt method, can be used to improve conditioning of the problem and enhance the convergence behavior.

Potential improvements to the modified Newton-Raphson technique include the use of adaptive strategies for updating the derivative approximation. Instead of using the damping factor, the method can be modified to update the derivative approximation based on the local behavior of the function. This adaptive approach can help improve the convergence rate and reduce the sensitivity to the initial derivative. Another possible enhancement is the combination of modified Newton-Raphson technique with other iterative techniques, such as secant technique or bisection technique. These hybrid techniques can leverage the strengths of different techniques to achieve faster convergence and improved robustness.

Conclusion

This paper presented a comparative analysis of modified Newton-Raphson technique with other iterative techniques for numerical simulation of nonlinear equations. The study evaluated the performance of modified Newton-Raphson technique, the standard Newton-Raphson technique, the secant technique, and the fixed-point iteration technique using benchmark problems and real-life applications from fluid dynamics, heat transfer, and structural mechanics as shown in table 3-5. The results demonstrated that the modified Newton-Raphson technique consistently outperformed the other iterative methods in terms of convergence rate and computational efficiency. The technique required the least number of iterations and exhibited the shortest execution time for all considered problems. The standard Newton-Raphson method closely followed, with slightly higher iterations and execution time, while the secant method ranked third in performance. The fixed-point iteration method showed the slowest convergence and the longest execution time as shown in table 2.

In terms of accuracy, all methods provided satisfactory results, with maximum absolute errors in the range of 1e-9 to 1e-5. The modified Newton-Raphson technique and standard Newton-Raphson technique exhibited comparable accuracy, with the modified version slightly outperforming the standard one. The secant method and the fixed-point iteration method yielded higher maximum absolute errors compared to the Newton-Raphson-based methods. The superior performance of the modified Newton-Raphson technique can be attributed to its use of an approximation of the derivative, which reduces the computational cost while maintaining a high convergence rate. The method's faster convergence and higher computational efficiency make it an attractive choice for solving complex nonlinear problems encountered in various fields of science and engineering.

However, the modified Newton-Raphson technique has limitations, such as its sensitivity to the initial derivative approximation and its performance in the presence of multiple roots or poorly conditioned problems. Potential improvements to the method include the use of adaptive strategies for updating the derivative approximation, the combination with other iterative techniques, and the exploitation of parallel computing architectures. Future research directions may include the extension of the comparative analysis to a broader range of nonlinear equations and development of more advanced numerical simulation techniques that can handle the limitations of the modified Newton-Raphson method. The integration of machine learning methods, such as deep learning or reinforcement learning, with numerical simulation methods could also be explored to enhance the efficiency and robustness of nonlinear equation solvers.

In conclusion, the modified Newton-Raphson technique demonstrated superior performance compared to other iterative techniques for the numerical simulation of nonlinear equations. Its faster convergence and higher computational efficiency make it a valuable tool for solving complex problems in science and engineering. With further advancements and improvements, the modified Newton-Raphson method has the potential to become a go-to technique for tackling nonlinear equations in various domains.

Abbreviations

Nil.

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Author Contributions

Inderjeet: Methodology, Software, Writing-Original Draf, Rashmi Bhardwaj: Conceptualization, Supervision, Investigation, Writing-Review and Editing.

Conflict of Interest

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