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# Applications of Fractal Geometry in Geosciences: A Literature Review

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## Abstract

Fractal geometry has rehabilitated a scientific field hitherto little explored by researchers, which is the study of natural or artificial irregular objects and structures, which have long been considered mathematical monstrosities. The study of these objects and structures using classical geometrical approaches and usual Euclidian geometry tools cannot be effectively done due to their complex characteristics. To overcome this issue, the concept of fractal geometry coined by Benoît Mandelbrot has been used immensely and applied successfully to several scientific fields of study, particularly in characterizing complex geological events and processes. The present article reviews and summarizes the recent available scientific research papers that deal with the application of the fractal geometry approach in Geosciences. The investigation of the available literature permits the delineation of a total of seven principal topics related to Geosciences including fracture networks, the mining industry, properties of rocks and materials, hydrology and drainage networks, geothermal gradient and heat flow, earthquake and volcanic activities, as well as the study of remote sensing images. Several methodologies and tools for studying the different geological events and processes are proposed by researchers and provide important insights and the possibility of integrating information and quantifying geological events measured at different spatial scales.

**Keywords:** Drainage Networks, Earthquakes, Fractal Dimension, Mineral Exploration, Rock Properties.

## Introduction

In 1610, Galileo said that mathematics is the language of nature and that its symbols are points, straight lines, circles, triangles and other geometric figures. His thoughts then seemed to prove him right. However, three centuries later, the Franco-American mathematician Benoît Mandelbrot argued that clouds are not spheres, nor mountains cones, nor islands circles and that these objects cannot be described by the habitual Euclidean geometry and necessitating the use of fractal geometry (1). To better understand the theory of fractal geometry, consider the classic example of measuring the length of the Britain coastline presented by Mandelbrot, which takes up the work carried out by Richardson in 1961 (2). Mandelbrot demonstrated that the length of the coast of Brittany varies according to the precision of the measurement, that is to say, the smaller the measurement ruler (scale) used, the greater the length measured. Consequently, when the measurement ruler tends towards zero, we obtain a coastline of infinite length. For that reason, calculating the length of an irregular object such as the coastlines using the usual Euclidian geometry tools didn't make sense, because as the measurement scale became smaller, the more the results tended towards infinity. Indeed, to provide an absolute measurement, Mandelbrot proposed the fractal geometry leads to the notion of fractal dimension to quantify irregular objects. Mandelbrot created a linear relationship between the logarithm of the length of the coastline and the logarithm of the length of the ruler used for the measurement, and the slope of this log-log relationship provides the fractal dimension. In traditional Euclidean geometry, there are four topological dimensions: 0 dimensions for points, 1 dimension for straight lines, 2 dimensions for plane surfaces, and 3 dimensions for volumetric shapes like spheres and cubes. However, the fractal dimension has a non-integer value, such as 1.2 for a bumpy line or 2.7 for an irregular surface, etc. In general, the rougher the texture, the higher the fractal dimension value, and vice versa. Benoit Mandelbrot also explains that the fractal objects are "self-similar", that is to say that each portion of a fractal object reproduces the general form, at different scales of magnification.



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It signifies that the fractal element is statistically the same regardless of the scale of magnification it is looked at. For example, taking a mountainous region, whatever the scale considered, it represents a spatial organization of a variety of rocks, themselves composed of a set of stones of smaller size and so on. This process is repeated down to the smallest grain. Using this mathematical concept as a starting point, it would be possible to describe many complicated objects and structures that seem to be fractal over a variety of scales. Some of the most common examples of natural fractals include landscapes, mountains, islands, rivers, rocks, etc. (3). When we analyze these fractals under different scales, we can observe the property of self-similarity extends over some orders of magnitude but is not infinite. Looking for example at a mountain range, one can discern a pattern that seems to repeat itself. This structure is repeated in the same way at different scales with impressive regularity. The fractal objects created by nature are attributable to the set of processes that intervene in the formation and development of these objects: erosion, tectonic, natural stresses, etc. The discovery of fractal forms in nature constitutes a form of unsuspected universality until then, but thanks to the rise of computer sciences, we have begun to generate fractals that preserve infinite self-similarity and are said to be deterministic.

In a deterministic object, the property of self-similarity is stated by the fact that the fractal object is invariant by certain transformations such as amplification, translation, or rotation. These fractals are generally obtained by an iterative process. In other words, even zooming indefinitely, the structure of the object will never change. Following this approach, mathematicians are led to generate geometric sets with a logarithmic base that repeats indefinitely (deterministic fractals) from an initiator (initial state) and a generator (the recurrence operation). In this case, the similarity is repeated indefinitely, whereas the similarity of natural objects is only statistical, with infinity having no meaning in the reality of the objects around us.

## Calculation of the Fractal Dimension

To explain the process of computing fractal dimension, we use the scaling property called homothety in mathematics. Figure 1 below shows a line, a square, a cube, and each of which is magnified by a factor of 3 (3 self-similar pieces). The dimensions of these geometric figures are 1, 2, and 3 respectively, and after magnification, the length of the line is multiplied by $3 = 3^1$, the surface of the square is multiplied by $9 = 3^2$, and the volume of the cube is multiplied by $27 = 3^3$. As a result, the size of each figure has been multiplied by $3^d$ where d is the dimension.

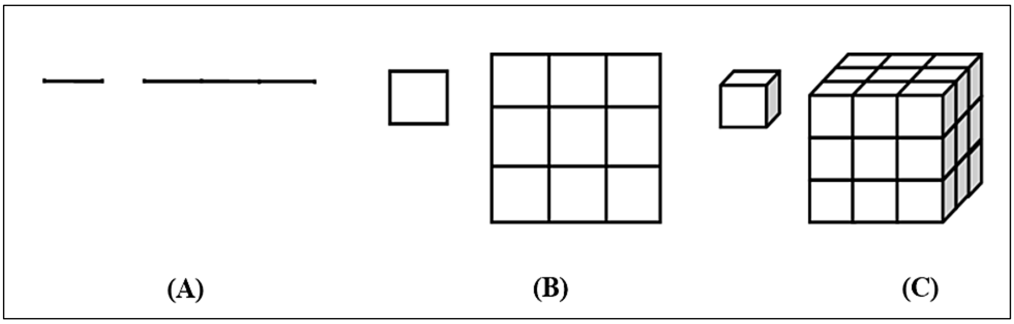


**Figure 1**: Examples of Regular Euclidean Geometric Figures with Dimensions (A): d=1, (B): d=2, and (C): d=3

There are many contributions of several famous mathematicians who have constructed objects using iterative schemes of a similar nature and that enabled the achievement of fractal dimension. Here are some that the reader may wish to explore:

## Von Koch Snowflake

Niels Fabian Helge Von Koch (1870-1924) is a Swedish mathematician. He proposed a fractal construction to which we gave his name: the Koch snowflake (Figure 2) (4). To achieve this, starting from an equilateral triangle, on each segment of length L, the central third of length r=L/3 is replaced by a "peak" formed of two segments of the same length. In other words, each segment is replaced by 4 segments 3 times smaller. So if we magnify three times, the length will be multiplied by 4 and we get four times the initial pattern. We can repeat the same process over and over again. Now how to determine the dimension of the Von Koch curve shown in Figure 2 below?

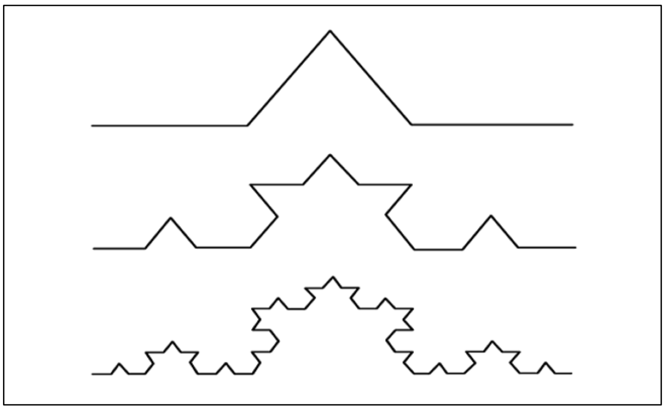

**Figure 2**: Illustration of Von Koch Curve (4)

The Von Koch curve is composed of exactly four similar parts. Therefore, if we magnify this curve three times, which is the scaling factor, *r*, we get four times the original shape. Therefore, its length is multiplied by 4, *N*, and its dimension d verifies the equation $3^d = 4$. In this view, this curve is neither dimension 1 nor dimension 2, because $3^1 < 4 < 3^2$. Thus, its dimension is a no-integer number between 1 and 2. So how do we find the exponent *d* in this case? Logarithms are required. The logarithm of a strictly positive number x verifies the property:

$$\mathrm{Log}(x^n) = n \log(x) \quad [1]$$

So in particular $\log(3^d) = d \log(3)$, and therefore since $3^d = 4$, we have $d \log(3) = \log(4)$, and thus the fractal dimension of the Von Koch curve is:

$$d = \frac{\log(N)}{\log(r)} = \frac{\log(4)}{\log(3)} = 1.262 \quad [2]$$

## The Cantor Set

Georg Cantor (1845-1918) is a German mathematician, known for his construction called the Cantor set, sometimes also called the no middle third set, Cantor comb, or Cantor dust (Figure 3). It was originally invented by the British mathematician H J S Smith but examined by Cantor in quite a different approach. The principle of the Georg Cantor set consists of dividing a straight line into three equal segments and then removing the middle third. Note that the extremities are kept in the set. Then, removes the central third of each of the new segments and so on until infinity, at each iteration the pattern is invariant (5).

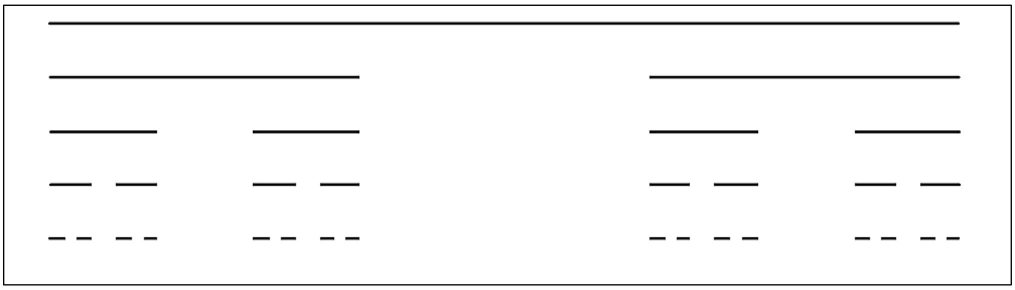

**Figure 3**: Illustration of the First Few Iterations of the Cantor Set (5)

Each of the two leaving segments is an exact replica of the previous segment but at 1/3 its scale, r. Hence, if the dimension is d, then $3^d = 2$. Therefore, using the logarithms we can obtain the fractal dimension using of the formula:

$$d = \frac{\log(N)}{\log(r)} = \frac{\log(2)}{\log(3)} = 0.631 \quad [3]$$

## The Construction of Sierpinski

Wacław Franciszek Sierpiński (1882-1969) is a Polish mathematician, known for his so-called Sierpiński construction (Figure 4) (6). The principle of this construction consists of taking a triangle (or any other regular polygon) and removing the central triangle. For each of the triangles thus formed, the central one is removed in the same way and the process continues indefinitely.

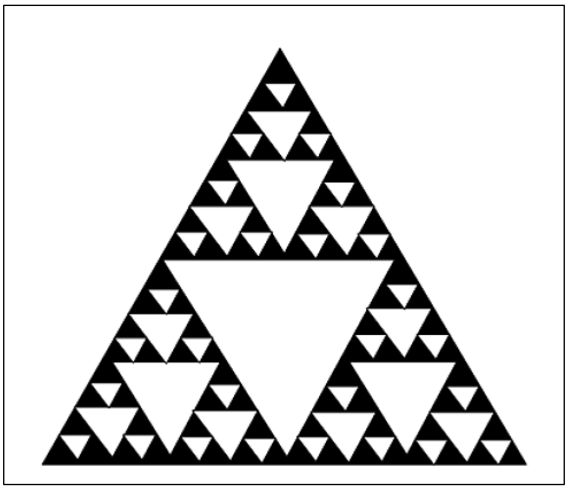

**Figure 4**: Illustration of the First Few Iterations of the Construction of the Sierpinski Triangle (6)

To determine the fractal dimension of the Sierpinski gasket (triangle), it must determine the scaling factor, *r*, and the number *N*. In this case, at each scale, each triangle is surrounded by 3 triangles of half the length of the triangle in the previous order. So the scaling factor r=2, and N=3. Therefore, we can find the fractal dimension by using the formula:

$$d = \frac{\log(N)}{\log(r)} = \frac{\log(3)}{\log(2)} = 1.585 \qquad [4]$$

## The Set of Gaston Julia

Gaston Maurice Julia (1893-1978) is a French mathematician specializing in functions of a complex variable. Julia presents his "Memory on the iteration of rational functions" and he discovers his sets called Julia sets (Figure 5) (7).

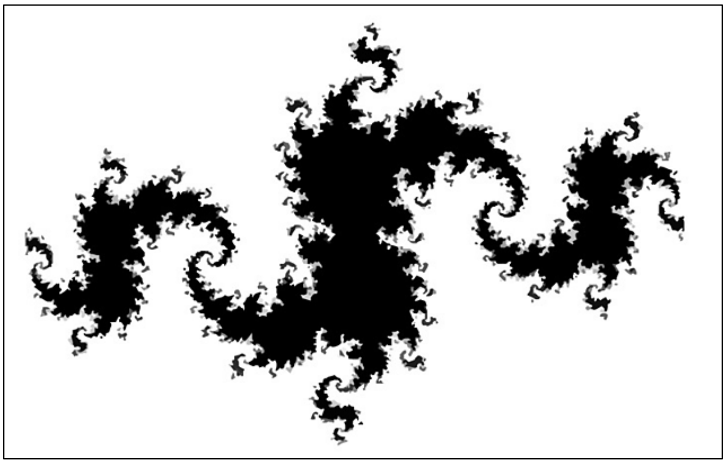

**Figure 5**: Illustration of Julia Construction (7)

Given two complex numbers, C and $Z_0$, defining the sequence ($Z_n$) by the recurrent relation:

$$Z_{n+1}=Z_n^2+c \qquad [5]$$

For a given value of C, the corresponding Julia set is formed by all initial values $Z_0$ for which the sequence is bounded. Note that there are infinitely many Julia sets since we can give any value to the constant C. Therefore, for any value of the parameter C, a Julia set exists.

## The Mandelbrot Set

Benoît Mandelbrot (1924-2010) was the godfather of fractal geometry. He is a Franco-American mathematician who showed the presence of fractals in a variety of objects and structures. Mandelbrot studied the expression of Gaston Julia $Z_{n+1}=Z_n^2+C$ and created his own set called "the Mandelbrot set" which is the set that encompasses all Julia sets (Figure 6). His work was honored in the scientific literature in his fundamental book “Les objets fractals, form, hasard et dimension” in 1975 (1), and then published in his second book “The Fractal Geometry of Nature” in 1982. It was at this time that the first applications of fractal geometry appeared.

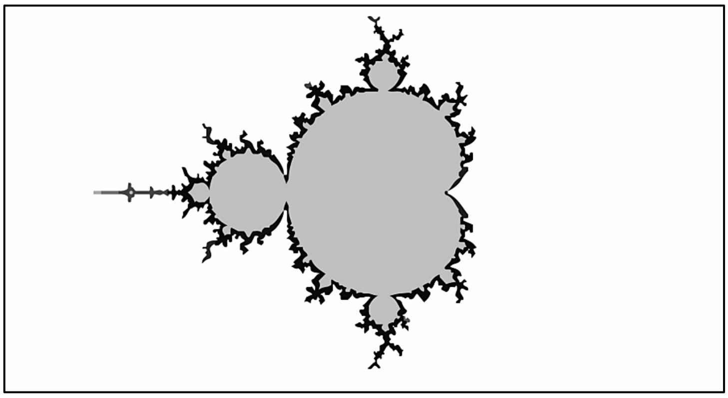

**Figure 6**: Illustration of Mandelbrot Construction (1)

By exploring the Mandelbrot set at different magnifications, one can find infinity of details. As with any fractal figure, at any magnification of the set we will find structures similar to those observed at lower magnifications, the size does not matter here.

There are different forms of self-similarity and scale invariance in Earth components and their related events. In the next sections, some of the recent scientific research discussing the application of fractal approach to geosciences is discussed.

# Discussion

## Application of Fractal Geometry to Geosciences

The study of geological phenomena and events (e.g. earthquakes, volcanoes, subduction, collision, mineral formation, etc.) that happened throughout the Earth's evolution is important both for recognizing the Earth's changes, as well as for in-depth knowledge of the composition of the Earth. However, due to the complexity of extreme geological events and temporal and spatial anomalies, classical geometrical approaches cannot be effectively used to quantitatively analyze geological events of complex characteristics. Hence, fractal methods can be used for quantitative description, classification, modeling, and prediction of such geological events. In the last decades, fractal geometry has become a very powerful classic tool used to analyze various geological events based on the property of self-similarity (2). This is very important for geoscientists to study such geological events on a small scale to predict the same phenomena on a larger scale. It is thus possible to analyze an immense zone by analyzing a single sample.

What are the main applications of fractal geometry in Geosciences, what are geological objects with a fractal character and how are they analyzed by the fractal approach? The answers to these questions were the result of a bibliographic research work that we will try to summarize in the following sections. This list of applications is not exhaustive; however, other examples have been discussed in the literature by many other researchers.

## Structure of Fracture Networks

Fractal geometry has been frequently used to analyze and assess the complexity of fracture networks and geological accidents, whether on a kilometer or millimeter scale. It promotes small secondary fractures linked to large fractures and shows that their spatial evolution and position are not uniform, but defined with a fractal hierarchy according to the power of the tectonic constraints reigning in the region. Examination of the internal structure of all fracture networks shows that it remains approximately identical to itself at all observation scales (8, 9). This structure can be quantified using the fractal dimension *d* as shown in Figure 7 below.

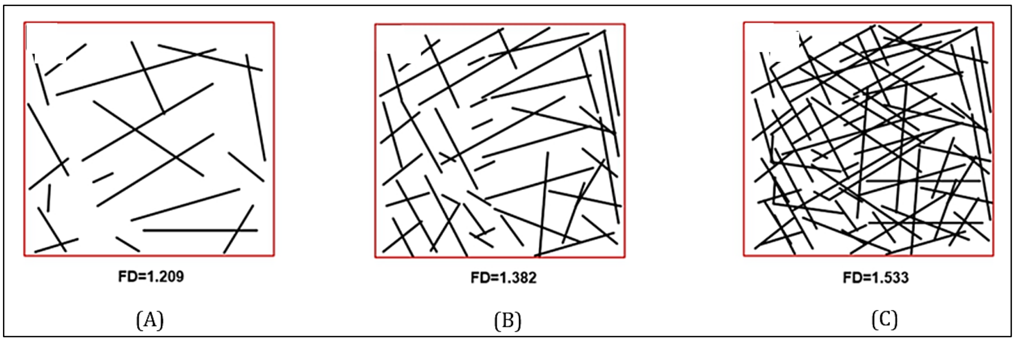


**Figure 7**: Calculated Fractal Dimension FD for 3 Testing Fracture Networks (A), (B), and (C)

The fractal behavior was used to analyze the spatial distribution of fractures. It was shown that the intersection between fractures belonging to the same network depends on the fractal dimension which can be determined by measuring the evolution of the average number of intersections per fracture. Indeed, the intersection of fractures increases when the fractal dimension decreases, and vice-versa (10). Moreover, the fractal dimension of the Qianhe Graben lineaments was estimated to assess the tectonic significance of active fractures and estimate the distribution characteristics of the lineaments (11). Furthermore, the fracture network on the crystalline bedrocks of the Rolvsnes granodiorite in southwestern Norway (Bømlo Island) was studied and showed that it is characterized by a scale-invariant distribution obeying a fractal law (12).

The fractal analysis was used to understand the spatiotemporal state of fractures in rock mass caused by mining from knowledge of its current state, as well as the laws and parameters that govern the variation of its behavior over time (13). It was also employed to examine the development of overburden fractures in coal mines to avoid disasters by observing the emerging law of overburden fractures. The researchers demonstrated that fracture formation began in the unmined region as a result of overstress (14), and was used to understand the complexity and development of the karst landscape. The local fractal dimension of the karst system was determined through the variogram of terrain profiles. This study demonstrates that the fractal dimension of 2.3, which is used as an upper limit for surface roughness based on surface fragility, is also an effective limit for karst terrains (15).

Regarding the multifractal approach, it has been used as an effective tool to describe the geometric complexity of 3D fracture networks and their relationship with the percolation and over-percolation stages. The results reveal that at the percolation stage, fracture orientations, lengths, and sizes exhibit positive correlations with multi-fractal singularity exponents. However, at the over-percolation stage, the lengths and sizes of the fractures become negatively related to the multi-fractal singularity (16).

## Mineral and Petroleum Industry

In addition to fracture networks, fractal geometry has found applications in the mineral and petroleum industry (17, 18). In this field, fractal geometry has been widely applied. Scientific researchers suggest that there are many problems in the mineral and petroleum sector that can be solved by fractal approaches. Firstly, it was applied to describe the complexity of geological structures. Researchers show that high complexity has high fractal dimension values and vice versa. Regions of high complexity have the potential to transport large volumes of hydrothermal fluid and provide physicochemical contrasts that are critical in the formation of large mineral deposits (19). Moreover, fractal approach has been used to study lineaments spatial distribution as indicators of fracture complexity in northeast Yunnan, China. The results provide insights for predicting the migration of ore-bearing fluids. Fractal contour maps were constructed using the fractal dimension of all lineaments and compared to mineral occurrences found in the study area. This comparison highlights that large deposits are consistent with high-value fractal dimensions (20). A similar attempt to delineate new mining prospects in central Morocco using fractal mapping was carried out and showed that the highest fractal dimension values were found around historical and/or recent metallic deposits (21).

The concentration area fractal analysis has been used to study stream silt to develop new tool chains to make Lithium (Li) exploration more effective and economical. The study used data from the pegmatite field of Fregeneda-Almendra in the Douro region of Portugal to identify pathfinder elements for Li and new target regions for Li. The fractal analysis technique was found to be efficient in reducing the number of regions of interest with over 75% accuracy when using Li-pegmatites maps. This can be better suited for prospect-scale prospecting as it allows the identification of four pegmatites containing Li mineralization in the Douro area (22). Also, local singularity and spectrum-area fractal approaches have been proposed as effective tools to assess stream sediments geochemistry to delineate arsenic (As) anomalies that correspond to porphyry copper (Cu) deposits and epithermal Au-Ag. As well as identify another fractal method, the zonality modeling and concentration-array (C-A) multivariate, was used to identify hypogene mineralization in the Kighal and Sungun porphyry Cu mines (northwest of Iran) (23). This fractal approach makes it possible to map potentially promising zones for lithogeochemical investigation in the Kighal area. The findings of this work demonstrate a relationship between Mo and Cu which are identical to elements associated with porphyry copper mines and can be used as an indicator of porphyry copper mineralization. Furthermore, the analysis of lithogeochemical data and isolation of anomalies using the zonality modeling and C-A multivariate fractal in the studied areas revealed that this approach is very effective in isolating anomalies and estimating the location of mineralization (24).

The fractal geometry has also been used to quantify the disturbed phosphate deposits in the Sidi Chennane mine (25, 26) by using various fractal approaches such as the box-counting method, mass-radius method (27), lacunarity and succolarity methods, multifractal spectra (28), and Triangular Prism Surface Area Method (29). These fractal analysis studies have shown that there is a significant correlation between the disturbed deposits and the corresponding fractal indices. It appears that the difference in the fractal indices can be used to distinguish two disturbed deposits of different rates. This has significant implications for distinguishing between phosphate deposits of low risk of disturbance and those of high risk. With these results, the researchers reveal that fractal analysis can be used as a critical concept in ranking potential phosphate deposits and thus improving the estimation of phosphate reserves.

In the petroleum industry (30), the fractal approach was used to design a fractal reservoir study model for optimizing petroleum and gas field management. The proposed model is expected to have numerous advantages in tackling reservoir study problems, improving the quality and flexibility of gas and petroleum reservoir studies, as well as a working time reduction (31). Another fractal model was suggested. It was a two-dimensional fractal stochastic model based on a Fourier transformation power spectrum technique used to estimate and characterize the spatial distribution of the potential gas resource in the volcanic reservoirs of the Songliao Basin, China. This model is used to predict resource potential under various exploration risk scenarios, as well as resource locations. The implementation of this model has provided information on the direction for future natural gas exploration (32).

In the following section, the application of fractal geometry to rock and soil materials is discussed as one of the most explored geological fields using the fractal approach.

## Structure of Rocks and Materials

Mandelbrot showed that the grains of rocks are organized according to fractal laws. By taking images at different scales of a granular surface, we will find that we are faced with the same initial image (33). This proves that fractal analysis can be used to understand the distribution of grains in rocks as well as their aggregation processes. This makes it possible to understand the mechanical behavior of the soil, which is considerably influenced by the grain size both at the microscopic and macroscopic scales. For example, in soil stability problems (study of foundations, retaining structures, etc.), it is necessary to know the resistance of the soil to deformation and rupture. In this context, fractal techniques can be used to analyze the granular materials employed in concrete and ceramics, in order to determine the degree of compactness and reduce the porosity of these construction materials (34).

Fractal geometry was used to study the change of pore structures and variations in the porosity of anthracite after electrochemical modification. The results showed that after the electrochemical modification, the porosity increased, the pore system became more regular, and the fracture system became more complex. These results were then presented to be used as a key guide for coalbed methane extraction (35). Moreover, the fractal approach has been used to examine the structural heterogeneity of pores in coal samples. The results reveal that the pores structure of the coals exhibited strong fractal features and that the fractal approach can be used to estimate the influence of pore structure heterogeneity on permeability. The findings contribute to establishing a relationship between pore structure and the behavior of gas flow in coal deposits (36).

Furthermore, researchers investigated the damage pattern of Lower Cambrian laminated shale rocks using the fractal dimension. The investigation was carried out on five distinct sets of shale with varying laminar dips that were examined at temperatures of 30°C, 60°C, and 90°C. The results reveal that crack shale growth is significant as a function of temperature. These findings were then evaluated using the fractal dimension, which shows an increase at 90°C and a decrease at 60°C, indicating the largest fracture growth in shale at 90°C (37). Another application research investigates the multi-scale morphological properties of crushed rock of sizes ranging from sand to gravel using fractal dimension as well as other numerical and computational methods. This research showed that as particle size decreases, the particle's corner becomes more rounded, the overall shape appears slightly angular, and the surface becomes smoother. The observed dependence of morphological descriptors on particle size has implications for studies using reconstructed particles of various sizes (38).

On the other hand, fractal theory was used to study the impact of water on the mechanical features and microstructures of coal for the stability design of coal pillar dams in the dry mining regions of western China. In this work, the fractal dimension was applied to scanning electron microscopy images of coal samples with 0-5 dry-wet cycles to evaluate the relationship between pore structure and microscopic morphology with mechanical damage. The damage variable, estimated by the fractal dimension, allows for a quantitative assessment of the microscopic damage of coal during the dry-wet cycle (39).

The analysis of rock fractures using fractal geometry may open a window to understanding and recognizing the characteristics of hydrology and drainage networks.

## Hydrology and Drainage Networks

The application of fractal geometry to hydrology and drainage networks started with the detection of chaos in rainfall data. Afterward, a wide range of issues related to various types of hydrological data, such as scaling and disaggregation, chaos detection and prediction, as well as the classification of watersheds were explored (40). Data examined include rainfall, rainfall-runoff, river flow, sediment transport, lake volume and level, and groundwater.

Hydrographic networks acquire a fractal tree structure that develops according to self-similarity to the extent that a linear segment can cover a planar space. The structure of any watershed remains almost the same at different scales. This fundamental structure of hydrographic networks was created naturally to optimize the distribution of water over a watershed. This structure has been used to understand the mechanics of watershed drainage and to determine some essential parameters (the length of the longest drain, the total length of the network, the number of endpoints, etc.) (41). The fractal tree approach has been used to establish the geomorphologic analysis framework over a river basin in Korea (42). Additionally, the fractal dimension was used to explain the geometric complexity of stream networks from distinct tectonic environments in Chile. Fractal analysis of stream networks carried out using the box-counting technique shows that the estimated fractal dimension varies significantly from one tectonic environment to another (43).

The fractal dimension method was used in the field of porous media to evaluate the residual saturation of non-aqueous phase liquids. The results of this study show that smaller pores in the medium correspond to a high fractal dimension, leading to high residual saturation of non-aqueous phase liquids; it is also found that micro-capillary pores composed of fine sand are the most critical controlling elements in the formation mechanism. This study shows that using the fractal dimension to evaluate the residual saturation of non-aqueous phase liquids is feasible (44). This method was also employed the fractal dimension to understand how water influences the mechanical behavior of coal rocks by measuring coal-splitting surfaces. Experimental results show that the fractal dimension of the splitting surfaces increases with increasing moisture content. Water allows a decrease of pre-existing coal mineral deficiencies. The splitting surface expands over pre-existing flaws, thus increasing the fractal dimension of the splitting surface (45).

From another side, fractal geometry was employed to study the sedimentation process of particles and sediments in water. It has been shown that there is an inverse log-log correlation between the sedimentation rate and the elapsed time over which the rate is assessed. The longer the sedimentation section, the more sedimentary breaks present. The sedimentation rates and the lengths of the intervening breaks have fractal-like features. The bigger breaks can be equated with the geologically more significant forms of sedimentary breaks, such as sequence boundaries and tectonic unconformities. Overall, the data on sedimentation rates and time scales is "fractal-like" (46).

In many cases, fracture networks can allow surface water to invade the permeable rocks and enter an aquifer, trickle down to great depths, be heated, and become geothermal water. Therefore, geothermal studies have also gained importance as one of the fields mainly investigated using fractal tools.

## Geothermal Gradient and Heat Flow

Fractal geometry has been widely and successfully applied in geothermal studies by various researchers. It was used to estimate the Curie temperature depth in the Mexican state of Coahuila using the "de-fractal" approach. The geothermal gradient was compared with the Bottom Hole Temperature data acquired from petroleum boreholes. This research revealed that the geothermal gradient in the study area is correlated to the Bottom Hole Temperature data. Furthermore, a correlation between the Bottom Hole temperature and an estimated geothermal gradient of the Curie temperature depth revealed that conduction is the main method of heat transmission (47).

Heat convection and conduction are critical to geothermal development. These two parameters are mainly influenced by the fluid flow in hot porous rocks. In this context, many researchers have studied heat conduction, and heat convection, based on the fractal approach. Such mathematical models of heat conduction were developed to investigate effective thermal conductivity characteristics in porous materials, based on the fractional Brownian motion (FBM). It has been found that pore structure has a significant impact on heat conduction. Thermal conductivity decreases as porosity increases, and vice versa. Even when porosity remains constant, the fractal dimensions of the porous material influence effective heat conductivity. Furthermore, it is shown that the heat conduction capacity of porous materials decreases with increasing fractal dimension (48). Moreover, some researchers propose a fractal thermal fluid coupling model to characterize flow behaviors and heat conduction in hot porous rocks using spatial derivatives and local fractional time. The fractal traveling wave transformation approach is used to solve this coupling equation. It has been found that the use of fractional parameters is required to accurately characterize the process of heat convection and conduction (49).

Moreover, fractal geometry was used to investigate high-temperature hydraulic fracturing at various injection flow rates using pre-crack granite specimens. The investigation revealed that the fractal fracture toughness decreases with decreasing injection flow rate and rising specimen temperature. Fractal fracture toughness is greater than linear fracture toughness, leading to increased fracture energy consumption due to micro-crack formation and mineral grain separation. These findings could give insight into the fracture process of hydraulically fractured thermal reservoirs at various injection flow rates (50). Also, the fractal dimension was employed to examine the cross-scale pores of granite after high temperatures. The results reveal that macropores and mesopores have major fractal properties, but micropores have the reverse. Furthermore, the fractal dimension of cross-scale pores is highly connected to the mechanical characteristics of the rock (51). Furthermore, researchers studied the impact of high temperature and water-cooling treatments on the dynamic fracture properties of granite. The computed fractal dimension of the fracture surface showed that porosity increased dramatically at heating temperatures over 400 °C. Furthermore, when the fractal dimensions of the fracture surface increased, both fracture energy and dynamic fracture toughness showed a power function drop trend (52).

In addition to all the applications discussed in the above sections, the following section will discuss the potential utility of fractal geometry to analysis data related to Earthquake activities and volcanic eruptions.

## Earthquake Activities and Volcanic Eruptions

Fractal geometry is a way to analyze and describe earthquakes. Seismologists prove that earthquakes are not isolated events. Seismicity has a fractal structure concerning time, magnitude, and space. Earthquakes most often occur in the form of a succession of shocks in a given location over several days, months, or years. If the main shock is a process of precursor events, it is almost systematically followed by a sequence of aftershocks. Indeed, the analysis of seismological data demonstrates a self-similarity of the time intervals between two earthquakes. That is to say that the time interval between two successive earthquakes depends on the recurrence of previous earthquakes, which may allow inferring the recurrence interval of an earthquake in a given area (53). Moreover, the fractal dimension was used to analyze the distribution of earthquake hypocenters (epicenters) in a given region, which could be essential for predicting large earthquakes (54), and studying the frequency of seismic precursors before major earthquakes in southern and Baja California. This study revealed a powerful relationship between low correlation fractal dimensions values and spatiotemporal clustering of earthquakes in the studied region. This work demonstrates the importance of fractal analysis in detecting seismicity patterns before significant seismic catastrophes. This should be supplemented by continuous monitoring to provide insights into seismic risk analysis and earthquake forecasting in unstable tectonic terrains (55).

An exploratory study was carried out to assess the potential relationship between the seismic moment rate and the fractal dimension on the island of Sumatra. The analysis shows a reasonable correlation. The high seismic moment rate model corresponds to the high fractal dimension. This might be extremely useful in the future for seismic mitigation in the study area as well as other similar region around the world (56). Another study used the fractal approach to develop a seismic wave equation and observed that earthquakes have a modified total energy that may be used to measure the fractal dimension's range. The results of this study show that slow earthquakes with fractal dimensions value $\alpha=1/2$, in particular, have a total energy E that is $E\alpha\approx10Ec$, where Ec is the conventional total energy (57).

Related to volcanic activity, various researches has been performed in terms of fractal geometry. More specifically, these researches have been aimed at the study of the distribution of volcanoes, the geometry of volcanic characters, the energy of explosive eruptions, the repose time of volcanoes, and so on. A study applied the fractal dimension (FD) to analyze the distributions of repose periods of volcanoes of various basaltic rocks over about 60 years. The work was carried out on the oceanic lithosphere of La Fournaise and Hawaii. The calculated fractal dimension is used to describe the strength of clustering of repose periods and shows that the more isolated the clusters correspond to the smaller values of FD. The main result obtained is the existence of a double regime with strong clustering of volcanoes for the short intervals defined by a low value of FD, and regular occurrence of volcanoes on large intervals defined by a high value of FD (58). A second study quantifies the complexity and variability of the shapes of volcanic particles for understanding fragmentation and transport processes associated with volcanic eruptions based on fractal geometry (pseudofractal dimensions). It was found that this fractal analysis process results in better discrimination between studied samples of sideromelane shards from Iceland (59). A third study use the fractal dimension to divide the topography of the volcano and its vicinity in the South China Sea into three parts: the steep waist part of the volcano, the mouth of the volcano, and the plane sea floor near the volcano (60). Another study was carried out to analyze the order clustering of volcanic events and the episodic discharge of material in the world using the fractal theory. Through this study, the researcher demonstrates that the volcano eruption rate exhibits spiky and episodic behavior and that the temporal structure of several clusters reveals that the tendency to clustering is self-similar: Volcano eruption sequences appear like impulsive noise.

Through this work, the researcher suggests that there must be a global mechanism necessary for the synchronization of spurts on Earth (61).

From another side, some scholars focus on using fractal geometry to study plate tectonics. The fractal analysis was used to demonstrate a major nonlinearity of the geometric distributions of plate tectonics elements concerning collision, subduction, rifting, orogenesis, transform faulting, etc. This research shows that the fractal characteristics of plate tectonic elements indicate that there is synergy between them and that they likely have a more profound significance for the Earth's geodynamic machine (62). Moreover, another research work supposes that plate tectonics might be one of the most natural manifestations of self-organized criticality. This is demonstrated when fractal growth processes and spatiotemporal power-law correlations are considered (63).

The following section discusses the latest application of fractal geometry in the context of this review article concerning the analysis of remote sensing images using fractal and multifractal techniques and tools.

### Analysis of Remote Sensing Images

With the rise of remote sensing in various geoscientific fields as a data acquisition method, fractal geometry has gained considerable interest and it seems to be an appropriate analysis tool because most of the images captured by satellites are discontinuous, fragmented, and complex. Fractal analysis has made it possible to examine the complex and erratic textures of a set of natural phenomena that have been considered serious research problems in remote sensing for several years. Several fractal methods have been used by researchers to quantify the complexity of remote sensing images (64, 65). Fractal models were also been performed to investigate the scaling behavior of geographical characteristics, which helps determine the optimal pixel resolution used in digital remote sensing images. Other studies have used local variation in fractal dimension values as a measure of texture to segment remote sensing images. The fractal dimension was used to determine an appropriate scale selection for image segmentation (RapidEye, Sentinel-2, and WorldView2) as a fully quantitative, easy, straightforward, robust, objective, and rapid tool to use for image segmentation (66). Moreover, the fractional Brownian motion (FBM) was used as an active classification method applied to Landsat satellite images of an area located east of Al-Kut city in Iraq. They observe that the capacity of FBM to study diverse regions on satellite images can achieve a classification score of 95% (67). Another study introduces a fractal extrapolation method that simulates the elevation in data-depleted regions, by making a realistic surface. The tests show that the proposed method is efficient enough to reconstruct a visually realistic surface (68).

Although it shows several advantages in characterizing various geological structures and the fractality of many geological structures was recognized, fractal geometry has many limitations and weaknesses, as it cannot describe all phenomena by the fact that not all geological structures are fractal (69). Moreover, using the single fractal dimension to depict the local scale properties and heterogeneity of the object in many fields such as the pores and fractures is inadequate (70). Furthermore, the application of fractals to the study of geological phenomena is an immature and developing field, and much of the theoretical rigor of fractal geometry has not yet been exploited and remains to be explained. Therefore, research in fractal theory needs to be accelerated to improve the application of fractal geometry to geosciences.

## Conclusion

At present, fractal geometry has become a new tool for qualitative and quantitative analysis applied to various geoscientific research fields, such as earthquakes, mining exploration, geothermic, etc. The present article reviews and summarizes the recent available scientific papers discussing the application of fractal geometry to geosciences. First, this review has provided a brief description of different categories of natural and determinist fractals and showed the importance of fractal analysis to identify and characterize natural and manmade structures. Afterward, various applications of fractal geometry methods to geosciences are reviewed from the aspects of fractal and multi-fractal characteristic extraction to evaluate and predict the properties of many geological events. Regardless of the essential utility of fractal methods in the field of geosciences, several measurement problems can be encountered. It should be noted that fractal analysis can be influenced by many factors such as the fractal analysis method used, the algorithms used, etc. Finally, many important geological topics are underappreciated by the reviewed works in this paper, and these gaps therefore should be addressed by future studies.

## Abbreviations

ED: Euclidean Dimension, FD: Fractal Dimension, FBM: Fractional Brownian Motion.

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Abderrahim Ayad: Conceived the study, analyzed the data, and wrote the manuscript.

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